# From SAT to SAT4J <br> Providing efficient SAT solvers for the Java platform 

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## Agenda

## What does SAT mean ?

Why is SAT successful ?

The SAT4J project

SAT4J : what about efficiency?

Pseudo Boolean Problems

Constraint Satisfaction Problems

MAXSAT

## The SAT problem

## Definition

Input: A set of clauses built from a propositional language with $n$ variables.
Output: Is there an assignment of the $n$ variables that satisfies all those clauses?

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Example

$$
\begin{gathered}
C_{1}=\{\neg a \vee b, \neg b \vee c\}=(\neg a \vee b) \wedge(\neg b \vee c) \\
C_{2}=C_{1} \cup\{a, \neg c\}=C_{1} \wedge a \wedge \neg c
\end{gathered}
$$

For $C_{1}$, the answer is yes, for $C_{2}$ the answer is no

$$
C_{1} \models \neg(a \wedge \neg c)=\neg a \vee c
$$

## Where are clauses coming from?

Suppose :
a I like free software
b I should start a free software project
c I should use a free software language
Then $C_{1}$ could represent the beliefs :

- $a \Longrightarrow b$ : If I like free software, then I should start a free software project.
- b c: If I start a free software project, then I should use a free software language.
What happens if I like free software and I do not use a free software language $(a \wedge \neg c)$ ? This is inconsistent with my beliefs. From $C_{1}$ I can deduce $a \Longrightarrow c$ : If I like free software, then I should use a free software language.


## $C_{2}$ SAT instance read by SAT4J (DIMACS)

$$
\begin{array}{llll}
\mathrm{p} & \operatorname{cnf} & 3 & 4 \\
-1 & 2 & 0 & \\
-2 & 3 & 0 & \\
1 & 0 & & \\
-3 & 0 & &
\end{array}
$$

Not really fun!

## SAT can be fun!

Fichier Edition Aide


## SAT is important in theory ...

- Canonical NP-Complete problem (Cook, 1971)
- Threshold phenomenon on randomly generated $k$-SAT instances (Mitchell,Selman,Levesque, 1992)


source : http ://www.isi.edu/ szekely/antsebook/ebook/modeling-tools-and-techniques.htm


## .. and in practice!

- Many problems can be solved using a reduction into SAT :

1996- Planning (SATPLAN,Blackbox)
1998- Software Specification (NitPick, Alloy)
1999- Bounded Model Checking, Equivalence checking, Formal Verification, etc.
2005- Pseudo Boolean constraints
2005- Constraints Satisfaction Problems

- SAT solvers are currently being used in production environments : Microsoft, Intel, IBM, Cadence, Synopsys, Valiosys, etc.
- Some people have fun with SAT : SuDoKu, Crosswords, Clue, etc.
- SAT technology is emerging in software engineering


## Examples of user applications

- The impact of satisfiability for Linux users package dependencies EDOS project, Opium bug finder e.g. SATURN
- The impact of satisfiability in software engineering Software specification Alloy4, Kodkod Feature modeling AHEAD Requirements analysis OpenOME Many more ...
- SAT solving can also be useful for solving security related applications (e.g. cryptanalysis or access control)!


## The SAT conference : www.satisfiability.org

- Workshops from Theoretical Computer Science or Artificial Intelligence

1996 Siena, Italy (TCS)
1998 Schloß Eringerfeld, Germany (TCS)
2000 Renesse, Netherlands (TCS)
2001 Boston, United States (IA)

- Yearly conference since 2002

2002 Cincinnati, United States
2003 Portofino, Italy
2004 Vancouver, Canada
2005 St Andrews, Scotland
2006 Seattle, USA

- Approximately 100 persons attend the conference each year
- SAT'07 will take place in Lisbon, Portugal


## The SAT Competition www.satcompetition.org

- The first competitions took place in the 90s :

1992 Paderborn, Germany
1993 2nd Dimacs challenge, United States
1996 Beijing, China

- Since 2002, it is a yearly event! Numerous participants :

200227 solvers
200330 solvers
200455 solvers
200543 solvers

- In 2006, there was a SAT Race (industrial friendly), not a SAT competition!
- Other competitions created after the SAT competition : 2003,2004 QBF

2005 QBF, PB, CSP, SMT, ...
2006 QBF, PB, CSP, SMT, MAX-SAT, ...

- The consequences are sometimes unexpected...


## The case of the termination competition

- solvers have one minute to prove that a term or string rewriting system terminates, e.g. :
INPUT: ( RULES b c -> a b b , b a -> a c b ) ANSWER: NO
Input system $R$ is not terminating since $R$ admits a looping reduction from bcaaca to aacabacbcaacabbb with 10 steps.
- huge success of the open source SAT solvers MiniSat and SatELite in the SAT 2005 competition
- Aprove, Jambox and Matchbox used them in 2006
- Results :

Aprove best for term rewriting systems (except the relative termination subcategory) and for logic programs
Jambox best for string rewriting systems and relat: termination of term rewriting

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## Why does SAT receive much attention currently?

- Most companies doing software or hardware verification are now using SAT solvers.
- Many SAT solvers are available from academia or the industry.
- SAT solvers can be used as a black box with a simple input/ouput language (DIMACS).
- A new kind of SAT solver was designed in 2001 (Chaff)
- algorithmic improvements
- new complexity/efficiency tradeoff
- designed with hardware consideration/limitation in mind


## Typical application for SAT : reachability analysis

## Definition

Given an initial state $s_{0}$, a state transition relation $S T$, a goal state $g$ and a bound $k$.
Is there a way to reach $g$ from $s_{0}$ using $S T$ within $k$ steps?
Is there a succession of states $s_{0}, s_{1}, s_{2}, \ldots, s_{k}=g$ such that
$\forall 0 \leq i<k\left(s_{i-1}, s_{i}\right) \in S T$ ?

- The problems are generated for increasing $k$.
- For small $k$, the problems are usually UNSATISFIABLE
- For larger $k$, the problems can be either SAT or UNSAT.
- Complete SAT solvers are needed!


## 1992 - Planning As Satisfiability

$$
\operatorname{PAS}(S, I, T, G, k)=I\left(s_{0}\right) \wedge \bigwedge_{i=0}^{k-1} T\left(s_{i}, s_{i+1}\right) \wedge \bigvee_{i=0}^{k} G\left(s_{i}\right)
$$

where :
$S$ the set of possible states $s_{i}$
I the initial state
T transitions between states
G goal state
$k$ bound
If the formula is satisfiable, then there is a plan of length $k$.

## 1997 - Software Model Analysis

$$
S M A(S, o p, p)=\exists s, s^{\prime} \in S \circ p\left(s, s^{\prime}\right) \wedge p(s) \wedge \neg p\left(s^{\prime}\right)
$$

where :
$S$ the set of possible states
op an operation
p an invariant
If the formula is satisfiable, then there is an execution of the operation that break the invariant.

Focus on encoding data structures so that the set of states $S$ could be structured

## 1999 - Bounded Model Checking

$$
B M C(S, I, T, p, k)=I\left(s_{0}\right) \wedge \bigwedge_{i=0}^{k-1} T\left(s_{i}, s_{i+1}\right) \wedge \bigvee_{i=0}^{k} \neg p\left(s_{i}\right)
$$

where :
S the set of possible states $s_{i}$
I the initial state
T transitions between states
$p$ is an invariant property
$k$ a bound
If the formula is satisfiable, then there is a counter-example reachable in $k$ steps.

Focus on translating LTL formulas into SAT

## Consequence in 2006

- Many Chaff-like solvers available in many languages.
- They can solve problems with millions of variables and clauses.
- SAT solvers are now designed to be embedded in other apps.
- Thanks to its standard input format, it is easy to test and use the latest SAT solvers available.

More and more applications are using SAT

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## The SAT4J project www.sat4j.org

- An open source library of Chaff-like solvers in Java
- Project started late 2003 as an implementation in Java of the MiniSAT specification.
- Library updated continuously with latest SAT technologies
- Efficiency validated during the SAT competitions (2004 and 2005) and the SAT Race 2006.
- Can also handle other kind of constraints :
cardinality $a+b+c+d \geq 3$
pseudo boolean $3 * a+2 * b+2 * c+d \geq 3$
- Built-in Constraint Satisfaction Problem (CSP) to SAT support (Participated to the First CSP competition in Summer 2005).
- Built in optimization problems support.
- Target easy integration in any Java software!


## Some SAT4J Users

Formal verification Kodkod project and Alloy4 (Daniel Jackson @ MIT)
Software engineering

- OpenOME (Yijun Yu et al @ U. Toronto)
- AHEAD (Don Batory @ U. Texas)
- FAMA (David Benavides @ U. Seville)

Semantic web Ontology matching in S-MATCH ( Fausto Giunchiglia, Pavel Shvaiko and Mikalai Yatskevich © U. Trento)

Constraints CONstraints ACQuisition (Christian Bessière, Rémi Coletta et al @ U. Montpellier)
Algorithm configuration Frank Hutter © UBC Other

- CROSSWORDS (Andy King and Colin Pigden)
- SUDOKU (Ivor Spence, U. Belfast)
- SAT4SATIN (Ibis group © Vrije)


## Use case 1 : OpenOME

http://www.cs.toronto.edu/km/openome/
OpenOME an Eclipse plugin for requirements engineering.
goal model to connect the user's high level requirements with the system's low level configuration items
preferences between goals : one goal is more important than another
expectations a goal needs to be satisfied to a certain degree

- Top-down reasoning propagates the expectations of high level goals downward to obtain the minimal number of low level goals that can fulfill the requirements.
- Top-down reasoning done with SAT4J


## Use case 2 : AHEAD tool suite

http://www.cs.utexas.edu/users/schwartz/ATS.html
Product line family of programs differentiated by features
Constraint Not all features are compatible
Safe Composition Avoiding type errors in the composed code.
AHEAD theory of software synthesis based on feature composition

SAT used to :

- debug feature models
- perform safe composition


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Product line family of programs differentiated by features
Constraint Not all features are compatible
Safe Composition Avoiding type errors in the composed code.
AHEAD theory of software synthesis based on feature composition
"Further, the performance of using SAT solvers to prove theorems was encouraging : non-trivial product-lines of programs of respectable size [40+ programs each with 35K Java LOC, ...] could be analyzed and verified in less than 30s."
Don Batory and Sahil Thaker, Safe Composition of Product Lines

## Use case 3 : Alloy

http://alloy.mit.edu

- 10 years old technology (Formerly Nitpick, 96)
- Followed the evolution of SAT solvers :
- Started with WalkSAT/SATO
- Then RELSAT/SATZ
- Took the Chaff wave
- Now uses MiniSAT
- Take advantage of new features in SAT solvers (e.g. unsat core)
- From the beginning in Java, relying on efficient C/C++ solvers (Java counterparts tried but abandoned)
- SAT4J allows a pure Java tool (still some problems with graph layout)


## Overview of SAT4J architecture : End User view



## SAT4J for Java programmers : basic use

```
ISolver solver = SolverFactory.newDefault();
solver.setTimeout(3600); // 1 hour timeout
Reader reader = new DimacsReader(solver);
try { // CNF filename is given on the command line
        IProblem problem = reader.parselnstance(args[0]);
        if (problem.isSatisfiable()) {
            System.out.println("Satisfiable !");
            System.out.println(reader.decode(problem.model()));
        } else {
            System.out.println("Unsatisfiable !");
        }
} catch (FileNotFoundException e) {
    catch (ParseFormatException e) {
    catch (IOException e) {
    catch (ContradictionException e) {
        System.out. println("Unsatisfiable (trivial)!");
} catch (TimeoutException e) {
    System.out.println("Timeout, sorry!");
}
```


## SAT4J for Java programmers : iterate over models

```
ISolver solver = SolverFactory.newDefault();
Modellterator mi = new Modellterator(solver);
solver.setTimeout(3600); // 1 hour timeout
Reader reader = new InstanceReader(mi);
try {// filename is given on the command line
    boolean unsat = true;
    IProblem problem = reader.parselnstance(args[0]);
        while (problem.isSatisfiable()) {
            unsat = false;
            int [] model = reader.decode(problem.model()));
            // do something with each model
        }
        if (unsat)
                            // do something for unsat case
} catch (FileNotFoundException e) {
        [...]
catch (ContradictionException e) {
        System.out.println(" Unsatisfiable (trivial)!");
} catch (TimeoutException e) {
    System.out.println("Timeout, sorry!");
}
```


## Overview of SAT4J architecture : Power User view



## SAT4J search visualization option



## SAT4J search visualization option



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## Efficient SAT solving with SAT4J

- Many possible solver configurations available (>20)
- No real benchmarking of solvers made for the previous competitions
- SAT Race is special : qualification stage
- Testsets are provided for the race (50 benchmarks)


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Choosing best configuration for the race

## First trial : test everything

| Solver | Solved | SAT | UNSAT | Time | Out Of Memory |
| :--- | ---: | ---: | ---: | ---: | ---: |
| MiniLearning | 36 | 13 | 23 | 319 m 31.771 s | 1 |
| MiniLearningHeap | 33 | 10 | 23 | 318 m 25.009 s | 5 |
| MiniLearningHeapEZSimp | 36 | 12 | 24 | 283 m 8.689 s | 3 |
| MiniLearning2 | 33 | 10 | 23 | 389 m 43.783 s | 0 |
| MiniLearning2Heap | 36 | 13 | 23 | 299 m 1.987 s | 0 |
| MiniLearning23 | 26 | 12 | 14 | 437 m 10.415 s | 0 |
| MiniLearningCB | 19 | 8 | 11 | 482 m 26.646 s | 1 |
| MiniLearningCBWL | 27 | 8 | 19 | 402 m 44.606 s | 1 |
| MiniLearning2NewOrder | 33 | 13 | 20 | 367 m 30.035 s | 0 |
| MiniLearningPure | 30 | 8 | 22 | 388 m 40.632 s | 1 |
| MiniLearningCBWLPure | 27 | 8 | 19 | 416 m 51.705 s | 1 |
| MiniLearningEZSimp | 35 | 12 | 23 | 309 m 29.826 s | 1 |
| MiniLearningNoRestarts | 31 | 10 | 21 | 379 m 31.950 s | 3 |
| ActiveLearning | 34 | 11 | 23 | 318 m 59.267 s | 1 |
| MiniSAT | 33 | 11 | 22 | 333 m 8.418 s | 1 |
| MiniSATNoRestarts | 30 | 9 | 21 | 377 m 36.427 s | 1 |
| MiniSAT2 | 33 | 10 | 23 | 377 m 36.427 s | 0 |
| MiniSAT23 | 25 | 11 | 14 | 437 m 44.362 s | 0 |
| MiniSATHeap | 33 | 10 | 23 | 298 m 31.360 s | 0 |
| MiniSAT2Heap (default) | 36 | 13 | 23 | 297 m 49.641 s | 5 |
| MiniSAT23Heap | 24 | 11 | 13 | 430 m 59.189 s | 1 |
| Relsat | 22 | 6 | 16 | 417 m 22.977 s | 2 |
| Backjumping | 10 | 7 | 3 | 621 m 9.357 s | 7 |

## Some competitors

| Solver | Solved | SAT | UNSAT | Time |
| :--- | ---: | ---: | ---: | ---: |
| MiniSat 1.14 | 38 | 12 | 26 | 230 m 56.139 s |
| zChaff 2004.11.15 | 34 | 9 | 25 | 368 m 26.901 s |
| Siege_v4 | 45 | 16 | 29 | 186 m 36.902 s |
| SatELite (not GTI) | 32 | 10 | 22 | 350 m 3.909 s |

## Second trial : change memory management

| Solver | $\#$ | SAT | UNSAT | Time | OOM |
| :--- | ---: | ---: | ---: | ---: | ---: |
| MiniLearning | 35 | 12 | 23 | 326 m 10.754 s | 1 |
| MiniLearningHeap | 34 | 11 | 23 | 317 m 48.771 s | 5 |
| MiniLearningHeapEZSimp | 37 | 12 | 25 | 277 m 16.920 s | 4 |
| MiniLearning2 | 33 | 9 | 24 | 363 m 50.988 s | 0 |
| MiniLearning2Heap | 37 | 13 | 24 | 279 m 29.925 s | 2 |
| MiniLearning2NewOrder | 35 | 12 | 23 | 360 m 2.050 s | 0 |
| MiniLearningHeap | 35 | 11 | 24 | 313 m 51.323 s | 1 |
| Activelearning | 33 | 12 | 23 | 332 m 2.813 | 1 |
| MiniSAT | 34 | 11 | 23 | 331 m 54.472 s | 1 |
| MiniSAT2 | 34 | 10 | 24 | 348 m 58.966 s | 0 |
| MiniSAT23 | 33 | 10 | 23 | 354 m 26.744 s | 0 |
| MiniSATHeap | 35 | 12 | 23 | 291 m 13.961 s | 5 |
| MiniSAT2Heap | 36 | 12 | 24 | 294 m 48.496 s | 3 |
| MiniSATHeapEZSimp | 37 | 12 | 25 | 296 m 24.180 s | 3 |

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| MiniSAT | 34 | 11 | 23 | 331 m 54.472 s | 1 |
| MiniSAT2 | 34 | 10 | 24 | 348 m 58.966 s | 0 |
| MiniSAT23 | 33 | 10 | 23 | 354 m 26.744 s | 0 |
| MiniSATHeap | 35 | 12 | 23 | 291 m 13.961 s | 5 |
| MiniSAT2Heap | 36 | 12 | 24 | 294 m 48.496 s | 3 |
| MiniSATHeapEZSimp | 37 | 12 | 25 | 296 m 24.180 s | 3 |
| MiniLearningHeapExpSimp | 42 | 14 | 28 | 297 m 32.545 s | 0 |

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| MiniSATHeapEZSimp | 37 | 12 | 25 | 296 m 24.180 s | 3 |
| MiniLearningHeapExpSimp | 42 | 14 | 28 | 297 m 32.545 s | 0 |
| Release 1.7, Java 6 RC | 42 | 14 | 28 | 276 m 31.717 s | 0 |

heap/array Heap based heuristics are definitely better for those benchmarks
$2 / 3$ specific binary data structures are helpful in some cases, better for solving satisfiable benchmarks.

Reason Simplification helps for solving UNSAT benchmarks. Expensive reason simplification from MiniSAT 1.14 is the best option for the SAT Race.
learning Filtering learnt clauses preserve efficiency.
memory management is the weakest part of SAT4J : despite a regular cleanup of the learnt clauses, the solver runs out of memory after a while.

## Are solvers in SAT4J state-of-the-art?

+ MiniSAT is currently the best available open source SAT solver (C++) : SAT4J started as a Java implementation of the original MiniSAT
+ SAT4J is a mature software (almost 3 year old) : core library has been fine tuned over the years
+ SAT4J is updated regularly with latest proven successful techniques
+ Java VMs are more and more powerfull : Java 6 VM will provide 20\% speedup for free
- Preprocessing available in MiniSAT 2.0 is not available in SAT4J
- SAT4J is designed for flexibility : fastest SAT solvers reimplement everything from scratch for heavy tuning!


## Results of SAT4J during the SAT Race 2006



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## Linear pseudo boolean constraints : definitions

- boolean variables $x_{i}$, truth value $\in\{0,1\}$.
- $\overline{x_{i}}=1-x_{i}$.
- General form :

$$
\sum_{i} a_{i} \cdot x_{i} \triangleright k
$$

where $a_{i}$ and $k$ are constants (integer or real) and $\triangleright \in\{=,>, \geq,<, \leq\}$.

- $k$ is called the degree of the constraint.
- Example: $3 x_{1}-4 x_{2}+7 \overline{x_{3}}-x_{4} \leq 2$


## Specific cases

Clauses and cardinality constraints can be seen as special cases of linear pseudo boolean constraints.

- $x_{1} \vee x_{2} \vee \ldots x_{n}$ translates to
$x_{1}+x_{2}+\ldots+x_{n} \geq 1$
- atleast $\left(k,\left\{x_{1}, x_{2}, \ldots, x_{n}\right\}\right)$ translates to

$$
x_{1}+x_{2}+\ldots+x_{n} \geq k
$$

- $\operatorname{atmost}\left(k,\left\{x_{1}, x_{2}, \ldots, x_{n}\right\}\right)$ translates to $\overline{x_{1}}+\overline{x_{2}}+\ldots+\overline{x_{n}} \geq n-k$.


## Resolution on clauses $=$ cutting planes on LPBC

$$
\begin{array}{cc} 
& \sum_{i} a_{i} \cdot x_{i} \geq k \\
\text { cutting planes: } & \sum_{i} a_{i}^{\prime} \cdot x_{i} \geq k^{\prime} \\
& \begin{array}{l}
\sum_{i}\left(\alpha \cdot a_{i}+\alpha^{\prime} \cdot a_{i}^{\prime}\right) \cdot x_{i} \geq \alpha \cdot k+\alpha^{\prime} \cdot k^{\prime} \\
\text { with } \alpha>0 \text { and } \alpha^{\prime}>0
\end{array} \\
&
\end{array}
$$

- we may form a combination which doesn't eliminate any variable.
- one single linear combination may eliminate more than one variable.


## Resolution on clauses $=$ cutting planes on LPBC

$$
\begin{array}{cc} 
& \sum_{i} a_{i} \cdot x_{i} \geq k \\
\text { cutting planes: } & \sum_{i} a_{i}^{\prime} \cdot x_{i} \geq k^{\prime} \\
\hline \sum_{i}\left(\alpha \cdot a_{i}+\alpha^{\prime} \cdot a_{i}^{\prime}\right) \cdot x_{i} \geq \alpha \cdot k+\alpha^{\prime} \cdot k^{\prime} \\
\text { with } \alpha>0 \text { and } \alpha^{\prime}>0
\end{array}
$$

- we may form a combination which doesn't eliminate any variable.
- one single linear combination may eliminate more than one variable.

$$
\begin{array}{cc} 
& \begin{array}{c}
x_{1}+x_{2}+x_{3} \geq 4 \quad 2 \overline{x_{1}}+2 \overline{x_{2}}+x_{4} \geq 3 \\
\\
\left(2 x_{1}+2\left(1-x_{1}\right)\right)+2+2 x_{3}+x_{4} \geq 8+3 \\
2 x_{3}+x_{4} \geq 7
\end{array}
\end{array}
$$

## SAT4JPseudo : Replacing resolution by cutting planes ...

- Using the CDCL framework proposed by GRASP
- With some improvements coming from Chaff (VSIDS, First UIP)
- Cutting planes are used during conflict analysis to generate an assertive constraint.
- Proposed first in Galena (Chai\&Kuehlmann 2003) and PBChaff (Dixon 2002/2004).
- Cardinality approach preferred to Full CP
- No management of integer overflow
- Solvers no longer developed


## The Pseudo Boolean evaluations

- Organized by Olivier Roussel and Vasco Manquinho in 2005 and 2006
- Uniform input format
- Independent assessment of the PB solvers
- Results freely available in details
- first comprehensive repository of benchmarks
- Various technologies used in 2006

|  | Mini- <br> Sat+ | SAT- <br> 4J | Pue- <br> blo | PBS | Bsolo | glpPB |
| :--- | :---: | ---: | ---: | ---: | ---: | ---: |
| Input | Clauses | LPBC | LPBC | LPBC | LPBC | LPBC |
| Inference | Res. | Full <br> C.P. <br> (boo- <br> lean) | Mixed | Mixed | Mixed | Full <br> C.P. <br> (real) |
| Optimization | L.S. | L.S. | L.S. | L.S. | B'n'B | Sim- <br> plex |

## Partial results of the PB05 evaluation

|  | Mi- <br> ni- <br> Sat+ | SAT- <br> 4J | Pue- <br> blo | PBS | Bsold |  |
| :--- | ---: | ---: | ---: | ---: | ---: | :--- |
| Decision problems | 43 |  |  |  |  |  |
| $(35)$ | 52 | 61 | 61 | 36 | UNS |  |
|  | 10 | 10 | 10 | 10 | 10 | UNS |
| Opt. Small | 176 | 120 | 160 | 133 | 159 | OPT |
|  | $(120)$ | $(226)$ | 182 | 0 | 180 | SAT |
| Opt. Medium | 0 | 2 | 0 | 0 | 0 | UNS |
|  | 24 | 19 | 34 | 33 | 28 | OPT |
|  | $(67)$ | 107 | 74 | 0 | 82 | SAT |
| Opt. Big | 103 | 85 | - | - | 90 | UNS |
|  | 26 | 3 |  |  | 9 | OPT |
|  | $(64)$ | $(171)$ |  |  | 83 | SAT |

## Partial results of the PB06 evaluation

|  | PB06 |  |  |  |  | Own |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{array}{r} \mathrm{Mi}- \\ \mathrm{ni}- \\ \text { Sat+ } \end{array}$ | $\begin{gathered} \text { SAT4- } \\ \text { C.P. } \end{gathered}$ | $\begin{array}{r} \text { Pue- } \\ \text { blo } \\ 1.4 \end{array}$ | $\begin{aligned} & \text { PBS } \\ & 4.1 \mathrm{~L} \end{aligned}$ | Bsolo | $\begin{gathered} \text { SAT4 } \\ \text { Res. } \end{gathered}$ |  |
| Decision pbms | 172 | 79 | 204 | 199 | 111 | 165 | UNS |
|  | 148 | 92 | 153 | 144 | 118 | 121 | SAT |
| Opt. Small | 43 | 54 | 37 | 29 | 40 | 35 | UNS |
|  | 405 | 357 | 385 | 352 | 409 | 367 | OPT |
|  | 250 | 303 | 323 | 0 | 280 | (267) | SAT |
| Opt. Medium | 03 | 04 | 04 | 05 | 06 | 05 | UNS |
|  | 9 | 9 | 15 | 0 | 7 | (9) | OPT |
|  |  |  |  |  |  |  | SAT |
| Opt. Big | 38 | 37 | - | - | 30 | 40 | UNS |
|  | 33 | 57 |  |  | 14 | 72 | OPT |
|  | 52 | 77 |  |  | 69 | 96 | SAT |

See http://www.cril.univ-artois.fr/PB06/results/ for details.

## Partial detailed results of the PB06 evaluation

|  | $\#$ | MSat+ | SAT4J | Pueblo | PBS | Bsolo | glpPB |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| SAT/UNSAT |  |  |  |  |  |  |  |
| pigeon | 20 | 2 | $\mathbf{2 0}$ | 13 | $\mathbf{2 0}$ | 2 | 20 |
| queens | 100 | $\mathbf{1 0 0}$ | 18 | 99 | $\mathbf{1 0 0}$ | $\mathbf{1 0 0}$ | $\mathbf{1 0 0}$ |
| tsp | 100 | 91 | 20 | $\mathbf{1 0 0}$ | 85 | 40 | 42 |
| fpga | 57 | 35 | 43 | $\mathbf{5 7}$ | 47 | 9 | 26 |
| uclid | 50 | $\mathbf{4 7}$ | 30 | 42 | 44 | 38 | 10 |
| OPT SMALLINT |  |  |  |  |  |  |  |
| minprime | 156 | $\mathbf{1 2 4}$ | 104 | 118 | 103 | 106 | 52 |
| red.-mps | 273 | 46 | $\mathbf{7 0}$ | 63 | 27 | 54 | 58 |
| OPT BIGINT |  |  |  |  |  |  |  |
| factor. | 100 | 14 | $\mathbf{5 2}$ | - | - | 7 | - |
| Ardal problems (one eq. constraint) |  |  |  |  |  |  |  |
| Ardal_1 | 12 | $\mathbf{1 0}$ | 2 | 0 | 3 | 2 | 0 |

See http://www.cril.univ-artois.fr/PB06/results/for details.

## Good cases for SAT4JPseudo

pigeon hole solvers using resolution cannot solve them. A nice way to check the inference engine of the solvers.
reduced mps Those benchmarks are composed of real LPBC, so solvers with CP capabilities have good results on them.
factorization SAT4J Heuristics was lucky on half of the benchmarks, because of the way it initializes the phase of the variables to branch on according to the objective function.

## Bad cases for SAT4JPseudo

TSP and Weighted Queens problems contributed by Gayathri Namasivayam for PB06. Much more clauses than cardinality constraints or PB constraints.
One typical example from the Queens problem :
SAT4J C.P. timeout at 1800 seconds after only 7 restarts for 2338 conflicts at 7 decisions/second
SAT4J Resolution 35 seconds after 16 restarts, 95829 conflicts at 3320 decisions per seconds

The difference lies in the conflict analysis procedure!

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## The difference lies in the conflict analysis procedure!

- Bad results of SAT4JPseudo during the evaluations do not mean Full C.P. approach is wrong : it depends of the implementation (c.f. PB2SAT @ PB05)
- Results heavily depend on the kind of benchmarks : many easy benchmarks make the comparison of solvers difficult.


## Agenda

What does SAT mean ?
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## From CSP to SAT

A CSP is a triplet ( $X, D, C$ ) such that
$X=\left\{X_{1}, X_{2}, \ldots, X_{n}\right\}$ is a set of $n$ variables
D is the domain function that maps to each variable $X_{i}$ its domain $D\left(X_{i}\right)$, i.e., the set of possible values for $X_{i}$.
$C=\left\{C_{1}, C_{2}, \ldots C_{m}\right\}$ is a set of constraints. Each constraint $C_{j}$ is a relation among the possible values for its variables.

## From CSP to SAT : naive version (Walsh, 2000)

Variables for each variable $X_{i}$, and each value $d_{j} \in D\left(X_{i}\right)$, a new propositional variable $p_{i, j}$ is created.
Domains for each variable $X_{i}$, a cardinality constraint specify that a single value can be selected from the domain : $\sum_{x} p_{i, x}=1$.
Forbidden Tuples (nogoods) Each forbidden tuple ( $x_{1}, x_{2}, \ldots, x_{k}$ ) is represented by a clause of length $k$ containing the negated proposition variables representing the values $x_{i}$.
Authorized Tuples (supports) Compute the complementary forbidden tuples and proceed as above.

## Example: 3-queens

- $X=\left\{X_{1}, X_{2}, X_{3}\right\}$
- $D\left(X_{i}\right)=\{1,2,3\} \forall i$
- Création des variables propositionnelles $v_{01}, v_{02}, v_{03}$
$v_{10}, v_{12}, v_{23}$
$v_{20}, v_{12}, v_{23}$
- Relation 1 (nogood) R1:

$$
(1,1)(1,2)(2,2)(2,1)(2,3)(3,3)(3,2)
$$

- Relation 2 (nogood) R2 :

$$
(1,1)(1,3)(2,2)(3,3)(3,1)
$$

- $C=\left\{C_{1}=R 1\left(X_{1}, X_{2}\right), C_{2}=R 2\left(X_{1}, X_{2}\right), C_{3}=R 1\left(X_{2}, X_{3}\right)\right\}$


## Constraints produced

- The domain definitions produce 3 cardinality constraints

$$
v_{01}+v_{02}+v_{03}=1, v_{11}+v_{12}+v_{13}=1, v_{21}+v_{22}+v_{23}=1
$$

- $C_{1}$ produces 9 binary clauses:

$$
\begin{aligned}
& \neg v_{01} \vee \neg v_{11}, \neg v_{01} \vee \neg v_{12}, \neg v_{02} \vee \neg v_{12}, \neg v_{02} \vee \neg v_{11}, \neg v_{02} \vee \\
& \neg v_{13}, \neg v_{03} \vee \neg v_{13}, \neg v_{03} \vee \neg v_{12}
\end{aligned}
$$

- $C_{2}$ produces 5 binary clauses:

$$
\neg v_{01} \vee \neg v_{11}, \neg v_{01} \vee \neg v_{13}, \neg v_{02} \vee \neg v_{12}, \neg v_{03} \vee \neg v_{13}, \neg v_{03} \vee \neg v_{11}
$$

- $C_{3}$ produces 9 binary clauses:

$$
\begin{aligned}
& \neg v_{11} \vee \neg v_{21}, \neg v_{11} \vee \neg v_{22}, \neg v_{12} \vee \neg v_{22}, \neg v_{12} \vee \neg v_{21}, \neg v_{12} \vee \\
& \neg v_{23}, \neg v_{13} \vee \neg v_{23}, \neg v_{13} \vee \neg v_{22}
\end{aligned}
$$

## Results of the first CSP competition (binary constraints)


source : http ://cpai.ucc.ie/05/CallForSolvers.html

## binary/n-ary

|  | SAT-based |  |  | Dedicated |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | biere | dleberre | roussel | dm6 | dongen | lecoutre |
| non binary constraints (147 benchmarks) |  |  |  |  |  |  |
| Solved | 26 | 52 | 50 |  | 70 | 97 |
| Time | 262 | 2425 | 1952 | - | 2337 | 8031 |
| binary constraints (922 benchmarks) |  |  |  |  |  |  |
| Solved | 377 | 739 | 769 | 822 | 818 | 759 |
| Time | 19894 | 15859 | 8070 | 12679 | 13642 | 18460 |

Selection of solvers that participated to the CSP05.
source : http ://cpai.ucc.ie/05/CallForSolvers.html

## From CSP to SAT : support version (Gent,2002)

Replace the translation of authorized binary tuples by constraints preserving arc consistency.

- For each set of authorized binary tuple like

$$
C=\left\{\left(a, b_{1}\right),\left(a, b_{2}\right), \ldots,\left(a, b_{n}\right)\right\}
$$

- Create a clause $\neg a \vee b_{1} \vee b_{2} \vee \ldots \vee b_{n}$
- Needed in both directions : for $a$, but also for $b_{i}$.
- For values not appearing in the constraints, unit negative clause!

Advantage Forbidden tuple computation no longer needed!
Drawback Produced clauses are no longer binary
Limited to binary constraints

## Naive/support comparison

On some benchmarks, the difference is obvious :

|  | Naïve (s) | Translation time (s) | Support (s) |
| :---: | ---: | ---: | ---: |
| hanoi3 | 1 | $<1$ | 1 |
| hanoi4 | 18 | 1 | 2 |
| hanoi5 | 731 | 33 | 2 |
| hanoi6 | - | 1840 | 7 |
| hanoi7 | - | - | 22 |

qk1 benchmarks (18 instances)
naïve no instance solved with 10 mn TO each support all solved (UNSAT) in less than $2 m n$

## January 2006 results (CRIL internal)

|  | SAT4J | Dedicated 1 | Dedicated 2 |
| :--- | ---: | ---: | ---: |
| non binary constraints (186 benchmarks) |  |  |  |
| UNSAT | 27 | - | 28 |
| SAT | 61 | - | 125 |
| binary constraints (2031 benchmarks) |  |  |  |
| UNSAT | 842 | 1004 | 995 |
| SAT | 760 | 840 | 827 |

## January 2006 results (CRIL internal) : non random

|  | SAT4J | Dedicated 1 | Dedicated 2 |
| :--- | ---: | ---: | ---: |
| non binary constraints (150 benchmarks) |  |  |  |
| UNSAT | 27 | - | 28 |
| SAT | 48 | - | 108 |
| binary constraints (1041 benchmarks) |  |  |  |
| UNSAT | 400 | 386 | 396 |
| SAT | 560 | 536 | 536 |

## September 2006 : Second CSP competition, first stage

|  | SAT4J | Abscon | BProlog | Buggy |
| :--- | :--- | :--- | :--- | :--- |


| non binary constraints (978 benchmarks) |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
| UNSAT | 68 | 77 | 46 | - |
| SAT | 273 | 429 | 379 | - |
| Total | 341 | 506 | 425 | - |
| binary constraints (2673 benchmarks) |  |  |  |  |
| UNSAT | 614 | 1053 | 598 | 1066 |
| SAT | 864 | 1290 | 858 | 1322 |
| Total | 1478 | 2343 | 1456 | 2388 |

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- For the first competition, constraints were given in extension.
- For the second competition, they can be given in intention.
- SAT-based encoding requires extensional form : it is sometimes impossible to generate it from the intensional form.


## Results of the second CSP competition (binary constraints)

Time to solve an instance (SAT/UNSAT answers, category 2-ARY)


| abscon $107 \mathrm{AC} \longrightarrow$ | buggy $2006 / 08 \longrightarrow$ |
| ---: | ---: |
| absoon $107 \mathrm{SAC} \longrightarrow$ | buggy ic $2006 / 08 \longrightarrow$ |
| sat4jCSP $1.7 \mathrm{meta} \longrightarrow$ | mistral $2006 / 08 \longrightarrow$ |

## Results of the second CSP competition ( n -ary constraints)



## Agenda

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## MAXSAT and the optimization framework

- Can use a linear search to solve optimization problems :

1. Find a solution
2. Evaluate its cost function
3. Add a new constraint to limit the search to better solutions
4. Repeat until no more solutions : latest one is optimal

- Allow solving MAXSAT by adding one selector variable per clause
- MAXSAT solver submitted to the first MAXSAT evaluation
- Results where pretty bad for MAXSAT (underlying SAT solver might not be appropriate). Binary Search and Linear Search solvers based on zChaff confirmed those bad results.
- Good results on one class of benchmarks in the weighted MAX-SAT category.


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## Conclusion

- SAT4J is a mature library of SAT solvers in Java
- The library allows easy integration of SAT technology into Java programs
- Additional features are provided :
- Pseudo Boolean solving
- CSP to SAT translation
- Optimization framework
- SAT4J evolves with SAT technology : new state-of-the-art features are integrated regularly.


## Future directions

- Improving Pseudo Boolean Solving
- Allowing reasoning on new And-Inverter Graph input
- Allowing manipulation of CSP constraints without grounding them
- Adding some MiniSAT 2.0 preprocessing techniques
- Improving user documentation and tutorials
- Separation of core SAT/PB/CSP code in next major release 2.0
- Release 1.7 is the first community driven release of SAT4J : more user-oriented features expected in the future
- Grid/Distributed Computing (Ibis and ProActive)

